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Exact superpotentials in $N = 1$ theories with flavor and their matrix model formulation

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Abstract

In this Letter we investigate the effective superpotential of an $N = 1$ $U(N_c)$ gauge theory with one adjoint chiral multiplet and N_f fundamental chiral multiplets. We propose a matrix model prescription in which only matrix model diagrams with less than two boundaries contribute to the gauge theory effective superpotential. This prescription reproduces exactly the known gauge theory physics for all N_f and N_c. For $N_f \leqslant N_c - 1$ this is given by the Affleck–Dine–Seiberg superpotential. For $N_f \geqslant N_c + 1$ we present arguments leading to the conclusion that the dynamics of these theories is also reproduced by the matrix model.

1. Introduction

In a recent series of papers [1–3], Dijkgraaf and Vafa have proposed a perturbative method for computing the effective glueball superpotential of several classes of $N = 1$ theories. This superpotential is essentially computed by summing over all the planar zero momentum Feynman diagrams of the theory. To better organize this computation in the case of fields in the adjoint representation of the gauge group, it is useful to express it as a matrix path integral with the potential given by the tree level superpotential of the original theory.

Although this duality was first obtained via a “string theory route” (building on previous work in [4–6]), it is purely a field theoretic duality, and very recently it has been derived within a gauge theoretic framework for $U(N_c)$ theories with adjoints [7]. Other related work on this duality has appeared in [8–14].

One of the most natural extensions of this duality is to theories with fields in the fundamental representation of the gauge group. The theory under consideration is supersymmetric QCD with gauge group $U(N_c)$ and N_f flavors coupled with a single chiral field in the adjoint representation.

One way to proceed is to represent the N_f fields in the fundamental representation as a $M_c \times M_f$ matrix while keeping $M_f/M_c = N_f/N_c$, and extend the matrix integral to contain such objects. Perturbing the resulting potential by mass terms for these fields allows one to integrate them out first and obtain an effective superpotential

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for the remaining adjoint fields. This method has been proposed in [15], and has yet to be tested against gauge theory predictions.

Another method [16] is based on the same integral, but uses the well-known fact that a quark loop in a large N_c graph corresponds to adding a boundary to the corresponding Riemann surface. If one has N_f flavors, graphs with multiple boundaries are weighted by powers of N_f/N_c. It is therefore possible to argue that, in the limit of small N_f/N_c, the matrix integral is dominated by two terms, one without quark loops, which is the typical Veneziano–Yankielowicz term and is proportional to the rank N_c of the gauge group, and the other coming from the diagrams with a single boundary. In this limit the contributions of multiple boundaries are suppressed. However, for N_f/N_c of order unity it is not a priori clear in this prescription that the diagrams with a higher number of boundaries are negligible.

This method was successfully used to compute in the matrix theory the effective superpotential of an $U(2)$ gauge theory with one flavor [16]. Very recently this analysis was extended to include theories with gauge group $U(N_c)$ and one flavor [17]. The superpotential evaluated at one of its extrema was presented as a series in the bare Yukawa coupling and was shown to agree up to 7th order in the expansion parameter with the one obtained in the gauge theory after integrating out the adjoint field, adding the Affleck–Dine–Seiberg (ADS) nonperturbative contribution and integrating out the quark fields.

The puzzle that triggered our analysis was the discrepancy between the apparent possibility of considering a larger number of boundaries in the matrix model analysis and the impressive agreement between gauge theory and the 1-boundary superpotential. Indeed, in the analysis of [16] diagrams with a higher number of boundaries are suppressed only by powers of $N_f/N_c = 1/2$. Since the gauge theory result is exact, and the agreement is exact up to factors of the order of $1/2^n$ where n is a large number, one cannot but suspect that this too good an agreement is due to the fact that all matrix model diagrams with more than a single boundary do not contribute at all.

We are therefore led to formulate a prescription in which only diagrams with less than two boundaries are relevant. This is accomplished by representing the gauge adjoint fields as $M \times M$ matrices, and the N_f fundamental fields as $M \times N_f$ matrices, and taking the large M limit. In the case when no fields in the fundamental representation are present, this reduces to the original Dijkgraaf–Vafa proposal. Our proposal is very similar in spirit to the one in [16]; the crucial difference is the automatic suppression of contributions of matrix diagrams with more than two boundaries.

The main puzzle raised by our prescription is that the matrix model does not distinguish between $N_f < N_c$ and $N_f > N_c$ while the gauge theory physics changes drastically.

In this Letter we first show that the effective superpotential obtained in the gauge theory by integrating out the adjoint field, adding the ADS superpotential, further perturbing by quark mass terms, and then integrating out the quarks *is in exact functional agreement* with the effective superpotential given by the matrix model graphs with less than two boundaries, after integrating out the glueball superfield S. Thus, the gauge theory superpotential (with the ADS component included) and the superpotential obtained in the matrix model are obtained from the same effective superpotential $W|_{\text{crit}}(\Lambda)$ by integrating in different fields.

We then argue that, for $N_f > N_c$, the gauge theory superpotential evaluated at its extrema is also reproduced by the matrix model.

The motivation behind our proposal was the impressive agreement between gauge theory and the matrix superpotential due to diagrams with less than two boundaries. However, our computations do not exclude the (unlikely) possibility that matrix diagrams with more than two boundaries are absent only in the particular case we consider. We will comment on alternatives at the end of our Letter.

2. Gauge theory

It was shown by Seiberg a long time ago that the requirement of holomorphy as well as the unbroken symmetries can be powerful allies for finding exact results regarding effective superpotentials in $N = 1$ supersymmetric theories. The simplest example of this sort is supersymmetric QCD. This theory contains an $U(N_c)$ vector multiplet as well as N_f pairs of quark fields, $\widetilde{Q}$, Q, with $\widetilde{Q}$ in the antifundamental of $U(N_c)$ and the fundamental of the $SU(N_f)$ flavor group, and Q in the fundamental of $U(N_c)$ and the antifundamental of a different $SU(N_f)$ flavor group.

The theory has no tree-level superpotential; the Lagrangian is thus:

$$\mathcal{L} = \int d^4\theta\, \mathrm{Tr}\big[\overline{Q} e^V Q + \overline{\widetilde{Q}} e^{-V} \widetilde{Q}\big] + \int d^2\theta\, \mathrm{Tr}\big[W^\alpha W_\alpha\big], \tag{1}$$

where the first and second traces are in flavor space while the last is in color space.

There are several different ways of assigning charges to the various fields and coupling constants present in this theory. One of them, which follows by requiring that the anomaly cancellation condition is satisfied by the physical fields alone, yields the following representations for the various fields, coupling constants and dynamical generated scale:

$$\begin{array}{rccccccccc} & SU(N_f) & \times & \widetilde{SU(N_f)} & \times & U(1)_i & \times & \widetilde{U(1)}_i & \times & U(1)_R \\ Q: & N_f & & 1 & & 1 & & 0 & & \frac{N_f - N_c}{N_f} \\ \widetilde{Q}: & 1 & & \overline{N}_f & & 0 & & 1 & & \frac{N_f - N_c}{N_f} \\ \Lambda^{3N_c - N_f}: & 1 & & 1 & & 1 & & 1 & & 0 \end{array} \tag{2}$$

Standard nonrenormalization theorems imply that there is no superpotential generated perturbatively. Nonperturbatively however, nonrenormalization theorems fail and a superpotential is generated [18,19]. Its form, up to numerical coefficients, is fixed completely by symmetries. For $N_f < N_c$ this can be argued in 3 steps.

(1) First, one notices that the moduli space can be described by the gauge invariant meson fields $X_i{}^j = \widetilde{Q}_i{}^a Q_a{}^j$. Requiring invariance under both flavor symmetry groups implies that the effective superpotential can be a function only of $\det X$.
(2) The requirement that the R-charge of the superpotential is 2 implies that the exponent of $\det X$ is $1/(N_f - N_c)$.
(3) Finally, on dimensional grounds, one has to add $\Lambda^{(3N_c - N_f)/(N_c - N_f)}$, where Λ is the dynamically generated scale of the theory.

The fact that the dynamical scale of the theory does not appear under a logarithm implies indeed that this superpotential is nonperturbatively generated. The coefficient of the resulting term can be explicitly computed for $N_c - N_f = 1$ ([19] for the $SU(2)$ gauge group and [20] for $SU(N_c)$) and is found to be equal to unity. For $N_c - N_f > 1$ the coefficient can be obtained by adding mass terms to the appropriate number of flavors and integrating them out. The result is the usual ADS superpotential [18]:

$$W_{\mathrm{dyn}} = (N_c - N_f)\left(\frac{\Lambda^{3N_c - N_f}}{\det X}\right)^{1/(N_c - N_f)}. \tag{3}$$

A slight deformation of the theory we are discussing is the inclusion of a tree level superpotential:

$$\frac{1}{2} a\, \mathrm{Tr}[\widetilde{Q} Q \widetilde{Q} Q] = \frac{1}{2} a\, \mathrm{Tr}\big[X^2\big]. \tag{4}$$

This term breaks the global $SU(N_f) \times \widetilde{SU(N_f)}$ to the diagonal subgroup. For consistency with the various symmetries of the original theory, the charge assignments for this new coupling constant are:

$$\begin{array}{rccccccc} & SU(N_f) & \times & U(1)_i & \times & \widetilde{U(1)}_i & \times & U(1)_R \\ a: & 1 & & -4 & & -4 & & 2\frac{2N_c - N_f}{N_f} \end{array} \tag{5}$$

This superpotential can be thought of as arising from integrating out a massive adjoint field with a cubic coupling with the quarks.

One can show that in this theory the only superpotential that can be generated is again the ADS one. Furthermore, adding mass terms to some of the quarks and integrating them out produces a new effective action which has the same form as the original one. This is achieved by keeping the coupling constant a fixed while taking the quark mass to infinity and keeping a certain combination of this mass and dynamically generated scale fixed. This procedure also yields the change in the dynamical scale to be

$$(\Lambda')^{3N_c-N'_f} = \Lambda^{3N_c-N_f} \det m, \tag{6}$$

where m is the mass of the quarks which were integrated out.

The central object $W|_{\mathrm{crit}}$ that contains all the information about the dynamics of the theory can be obtained by integrating out all the fields. The resulting function depends on the scale Λ_0[1] as well as on the coupling constant a and the masses of the quark fields.[2] Knowledge of this function allows one to reconstruct the full superpotential via Legendre transforms [18].

Without loss of generality we can integrate out all the quark fields at the same time, by introducing a mass matrix m proportional to the identity matrix. It is certainly possible to introduce hierarchical masses (and/or nondiagonal mass matrices) and integrate out one quark field at a time. However, the final result will be expressed only in terms of the scale Λ_0; the expression in terms of more general mass matrices can be trivially restored.

It is customary to write the superpotential in terms of the meson field $X = \widetilde{Q}Q$. Then, the superpotential deformed by mass terms is

$$W_{\mathrm{eff}} = m\,\mathrm{Tr}[X] - \frac{a}{2}\,\mathrm{Tr}\big[X^2\big] + (N_c - N_f)\left[\frac{\Lambda_{N_f}^{3N_c-N_f}}{\det X}\right]^{1/(N_c-N_f)}, \tag{7}$$

where Λ_{N_f} is the scale of the theory with N_f light fields. Since we assumed that the mass matrix is proportional to the identity matrix, it follows that, at the minimum of W_{eff}, X will have the same property. Thus, we can write from the outset

$$X = x\mathbb{1}_{N_f}. \tag{8}$$

By simple inspection of the superpotential above it is clear that explicitly finding its extrema for general numbers of colors and flavors is not an easy operation. The approach we take is to use the equation of motion for x to cast the superpotential into a simple form in which one still has to replace x by the solution of some algebraic equation.

The equation for x is simply:

$$m - ax - \frac{\Lambda_{N_f}^{(3N_c-N_f)/(N_c-N_f)}}{x^{(N_c)/(N_c-N_f)}} = 0. \tag{9}$$

It is convenient to introduce new dimensionless variables

$$y = \left[\frac{m}{\Lambda_{N_f}^{2k+1}}\right]^{1/k} x, \qquad \beta = \frac{a}{m}\left[\frac{\Lambda_{N_f}^{2k+1}}{m}\right]^{1/k} \equiv \frac{g^2}{m^2 M}\Lambda_0^3, \quad k \equiv \frac{N_c}{N_c - N_f}, \tag{10}$$

[1] The index denotes the fact that there are no more light fields in the theory.

[2] If one interprets the quartic tree level superpotential as arising from integrating out an adjoint field, then the coupling constant a is a function of the mass of the adjoint field and the strength g of the trilinear coupling $\mathrm{Tr}[\widetilde{Q}\phi Q]$; $a = g^2/M$.

where Λ_0 is the dynamical scale of the theory with no light fields.[3] In terms of these variables the equation of motion for x becomes

$$\beta y^{k+1} - y^k + 1 = 0, \tag{12}$$

or equivalently

$$y^{-k} = 1 - \beta y, \tag{13}$$

since $y = 0$ is not a solution of Eq. (12).

Now we use these equations to eliminate the quadratic term in W_{eff}. A small amount of algebra leads to the superpotential evaluated at its minimum in terms of the newly introduced variables y and k:

$$W|_{\text{crit}} = \frac{1}{2} N_f \Lambda_0^3 \left[y + \frac{k+1}{k-1} \frac{1}{y^{k-1}} \right], \tag{14}$$

where again y is a solution of Eq. (13).

This is the form that will be compared with the matrix model predictions. In the next section, using our modified extension of the Dijkgraaf–Vafa prescription to include fields in the fundamental representation, we will compute the value of the superpotential at its critical points and recover Eqs. (14) and (13) *for all N_c and N_f*. We will then present a gauge theoretic argument that (14) and (13) are also true for $N_f > N_c$.[4]

3. The matrix model

The prescription of Dijkgraaf and Vafa instructs that to compute the effective superpotential for the gaugino condensate, we have to compute the planar partition function for the matrix model with a potential which is the tree level superpotential of the $N = 1$ theory we are interested in. The original arguments covered theories which had fields in bi-fundamental representations of the gauge group.

Applied to a superpotential with a single critical point, this proposal yields the effective superpotential for the glueball superfield S by the following three steps:

First, one computes the contribution to the free energy due to planar diagrams, $\mathcal{F}_0$. This is accomplished by formally replacing the gauge theory fields with $M \times M$ matrices in the matrix model potential. Then one computes the path integral in the limit $M \to \infty$.

The second step is to identify the ’t Hooft coupling in the matrix model with the gauge theory glueball superfield. At this stage $\mathcal{F}_0$ becomes a function of S only.

The third and last step if to construct the gauge theory effective superpotential as

$$W_{\text{DV}} = N_c \frac{\partial \mathcal{F}_0}{\partial S} + \tau S. \tag{15}$$

The only gauge theory ingredients entering this relation are N_c—the number of colors, and τ—the bare coupling constant. There is no relation between N_c and the dimension of the matrices used in the matrix model computations; the first one is a parameter while the second one was identified with the glueball superfield.

[3] Indeed, by taking $N'_f = 0$ in Eq. (6) and using the definition of k it is easy to see that

$$\Lambda_0^3 = m \left[\frac{\Lambda_{N_f}^{2k+1}}{m} \right]^{1/k}. \tag{11}$$

[4] The case $N_c = N_f$ is problematic since the ADS superpotential does not admit a continuation to this point.

In the case of a trivial superpotential the only contribution to $\mathcal{F}_0$ comes from the normalization of the matrix path integral by the volume of the gauge group. In this case $W_{\rm DV}$ reproduces the Veneziano–Yankielowicz superpotential.

Attempts to extend this prescription to include fields in the fundamental representation of the gauge group were formulated in [15,16]. The idea is again to use the tree level superpotential as potential for the matrix model. In one proposal both the number of flavors N_f and the number of colors N_c are "promoted" to matrix model variables M_f and M_c such that $N_f/N_c = M_f/M_c$ [15]. In the other proposal none of them is [16]. In both cases, in the limit $N_f \sim N_c$ one cannot perform an expansion in the number of boundaries in the matrix model, which makes comparison with gauge theory difficult.

Here we propose that the matrix model variables are $M \times M$ matrices if the corresponding gauge theory fields are in the adjoint representation and $M \times N_f$ matrices if there are N_f fields in the fundamental representation in the gauge theory. Then, as in the original DV prescription, we identify the gauge theory glueball superfield with the 't Hooft coupling of the matrix model, i.e., $S = g^2 M$. As put forward in [16], the extended DV superpotential is the sum of the original one and extra contributions coming from matrix diagrams with boundaries:

$$W_{\rm DV} = N_c \frac{\partial \mathcal{F}_0}{\partial S} + \tau S + \mathcal{F}_{\rm boundaries}. \tag{16}$$

If one recalls that S was identified with the dimension of the matrices and examines the M dependence of the various terms in Eq. (16), it becomes clear that *only* diagrams with one boundary contribute in the planar (large M) limit. Indeed, Eq. (16) can be organized as an expansion in (N_f/M). Then, due to the derivative with respect to S, the first term and the contribution of diagrams with one boundary will be of the same order in an $1/M$ expansion. *All diagrams with two or more boundaries are suppressed by factors of* $1/M$ *and do not contribute in the large* M *limit.*

As in the original DV proposal, the part of $\mathcal{F}_0$ arising from the volume of the gauge group yields the Veneziano–Yankielowicz superpotential. This part is not modified by the inclusion of fields transforming in representations of flavor symmetry groups because these symmetries are only global.

For supersymmetric QCD with an adjoint field ϕ this leads to [15,16]:

$$\begin{aligned} e^{\mathcal{F}} &= \int D\phi\, DQ\, D\widetilde{Q}\, e^{-[\frac{1}{2}M_\phi \operatorname{Tr}[\phi]^2 + m \operatorname{Tr}[\widetilde{Q}Q] + g \operatorname{Tr}[\widetilde{Q}\phi Q]]} \\ &= \int DQ\, D\widetilde{Q}\, e^{-[m \operatorname{Tr}[\widetilde{Q}Q] - \frac{1}{2} a \operatorname{Tr}[\widetilde{Q}Q\widetilde{Q}Q]]} \end{aligned} \tag{17}$$

with $a = g^2/M_\phi$, as in the gauge theory discussion. The path integral above can be computed in both of its forms, using the analysis of [21]. Using the first form above and the analysis in [16] we can show that the contribution to the free energy of the matrix model in the large M limit is

$$\mathcal{F}_{\chi=1} = -N_f S\left[\frac{1}{2} + \frac{1}{4\alpha S}\left(\sqrt{1-4\alpha S} - 1\right) - \ln\left[\frac{1}{2} + \frac{1}{2}\sqrt{1-4\alpha S}\right]\right], \tag{18}$$

where $\alpha = a/m^2$. Then, according to our extended DV prescription, the superpotential is

$$\begin{aligned} W_{\rm DV} &= N_c S\left[1 - \ln\frac{S}{\Lambda^3}\right] + \mathcal{F}_{\chi=1} \\ &= N_c S\left(1 - \ln\frac{S}{\Lambda^3}\right) - N_f S\left[\frac{1}{2} + \frac{1}{4\alpha S}\left(\sqrt{1-4\alpha S} - 1\right) - \ln\frac{1}{2}\left(1 + \sqrt{1-4\alpha S}\right)\right], \end{aligned} \tag{19}$$

for all N_f and N_c.

As in the gauge theory case, we proceed by integrating out the massive fields. Various terms in the equation above imply that the gaugino fields acquired a mass and thus can be integrated out. Despite the complicated form of the superpotential, its extrema are given by a very simple equation:

$$N_c \ln \frac{S}{\Lambda^3} = N_f \ln \frac{1}{2}\left(1+\sqrt{1-4\alpha S}\right). \tag{20}$$

The first step is to use this equation to eliminate the logarithms in W_{eff}. Then, the critical point equation can be cast in a more useful form:

$$\sqrt{1-4(\alpha S)} = 2\left(\frac{(\alpha S)}{\alpha\Lambda^3}\right)^{k/(k-1)} - 1 \tag{21}$$

which in turn leads to the following expression for the superpotential at the critical points:

$$W|_{\text{crit}} = \frac{N_f}{2\alpha}\left[\frac{k+1}{k-1}(\alpha S) - \left(\frac{(\alpha S)}{\alpha\Lambda^3}\right)^{k/(k-1)} + 1\right]. \tag{22}$$

This equation does not appear similar to the corresponding one on the gauge theory side. It is nevertheless possible to relate them more closely. To this end we massage Eq. (21). A small amount of algebra combined with the observation that $S = 0$ is not a solution of Eq. (21), casts it in the following form:

$$1 - \left(\frac{(\alpha S)}{\alpha\Lambda^3}\right)^{k/(k-1)} = \left(\alpha\Lambda^3\right)\left(\frac{(\alpha S)}{\alpha\Lambda^3}\right)^{-1/(k-1)}. \tag{23}$$

Introducing the notation

$$w = \left(\frac{(\alpha S)}{\alpha\Lambda^3}\right)^{-1/(k-1)}, \tag{24}$$

Eq. (23) becomes

$$w^{-k} = 1 - \left(\alpha\Lambda^3\right)w \tag{25}$$

which is the same as Eq. (13).

In terms of this new variable the superpotential evaluated at its minimum is given by

$$W|_{\text{crit}} = \frac{1}{2}N_f\Lambda^3\left[\frac{k+1}{k-1}\frac{1}{w^{k-1}} + w\right], \tag{26}$$

where w must be replaced by a solution of Eq. (25).

Since w in the matrix model result as well as y on the gauge theory side are dummy variables, the results of the two computations agree provided that the matrix model scale Λ is identified with the gauge theory scale in the confining vacua as

$$\Lambda^3 = \Lambda_0^3 = (\det m)^{1/N_c}\Lambda_{N_f}^{(1+2k)/k}, \tag{27}$$

which is consistent with both theories not having massless fields. Furthermore, since the analysis above goes through if $N_f = N_c$ this seems to suggest that the same should hold in the gauge theory as well.

4. $N_f > N_c$

It is clear from Eq. (19), and it was also pointed out in [16] that the extended DV superpotential is insensitive to the range of N_c and N_f. It was suggested in [16] that this "problem" might be solved by including contributions from additional boundaries, which in the $N_f \to N_c$ limit would become important. The proposal we put forward in this Letter implies that such contributions are inexistent.

We have shown above that the 1-boundary matrix integral reproduces the expected gauge theory answer for *all* N_f and N_c for which the ADS superpotential holds. Moreover, this range can be extended to include also the case $N_f = N_c + 1$. Indeed, without introducing baryonic sources, the superpotential in this case is just the continuation of the ADS one [22] to this number of flavors.

The $W|_{\text{crit}}$ obtained in the matrix model always looks as if one had blindly started in gauge theory from an ADS-like superpotential, even for $N_f > N_c$. Our proposal implies that, even though for $N_f > N_c + 1$ no superpotential is generated in the beginning, once all quark fields are integrated out the superpotential evaluated at the critical point is still given by Eqs. (26) and (25). At first glance this might seem problematic. It is nevertheless possible to argue that there is no contradiction with the field theory analysis [23].

The argument is a slight generalization of the analysis in [23] and is based on Seiberg's duality together with the possibility of freely passing from electric variables to magnetic variables in the path integral.

In gauge theory, one can deform the theory by adding mass terms for all quark fields and then integrate them out. For $N_f \geqslant N_c + 2$ the gauge theory is strongly coupled and the correct description is given by its Seiberg dual. In this description we could add masses to the magnetic quarks and integrate them out, until we reach the theory with a completely broken gauge group. The superpotential for this theory is written in terms of the fields which are dual to the electric meson fields and, in the absence of baryonic sources, is just the ADS superpotential. At this point we dualize back to the electric theory which is now weakly coupled.

The results of the gauge theory analysis for $N_f = N_c + 1$, given by (14) and (13), can now be applied without reservations. Since the matching of scales when fields are integrated out is fixed by the renormalization group equations, it follows that the effective scale when all quarks are integrated out is given by Eq. (6) with the initial N_f and N_c [22,23].

Thus, by integrating out all fields in the fundamental representation, we obtain the same $W|_{\text{crit}}$ as if we blindly started with an electric ADS-like superpotential for $N_f \geqslant N_c + 2$ and integrated out all quark fields. This explains the apparently unnatural agreement between the matrix model results and the introduction of an ADS-like superpotential for $N_f \geqslant N_c + 2$.

5. Discussion

In this Letter we formulated an extension of the Dijkgraaf–Vafa proposal which includes fields in the fundamental representation. Basically, this proposal states that *only matrix model diagrams with less that two boundaries contribute to the* "*gauge theory–matrix model*" *duality*. We have tested this proposal by analyzing in detail supersymmetric QCD coupled to an adjoint chiral multiplet, both from the gauge theory and the matrix model perspective. We have found that, in the range of parameters where the Affleck–Dine–Seiberg superpotential is valid, it is reproduced exactly by the matrix model computation. Using Seiberg's duality we have then argued that this agreement persists even for $N_f > N_c$, when the gauge theory is strongly coupled. Therefore the gauge theory and the matrix model analysis described earlier in this Letter agree for all values of N_c and N_f.

Since for theories with $N_f \geqslant N_c$ the baryons acquire a nonvanishing expectation value, it would be interesting to repeat the analysis by including baryonic sources both in the matrix model and in the gauge theory. This would extend the number of arguments of $W|_{\text{crit}}$ and would provide a further test of the extension of the DV prescription proposed in this Letter.

It would also be interesting to see how our proposal would be implemented when the recent field-theoretic proof [7] of the initial DV proposal is extended to include fields in the fundamental representation. Another worthwhile endeavor would be finding the gauge theory interpretation of matrix model diagrams with more than one boundary. According to our proposal they are suppressed like the nonplanar diagrams in the original matrix model; it seems however likely that they do not correspond to gravitational corrections, given the difficulties with the geometric engineering of such theories.

As we pointed out in the Introduction, our computations do not exclude the (unlikely) possibility that matrix diagrams with more than two boundaries are only absent in the particular case we consider. We can imagine several possible reasons for this:

(1) The extension of the DV proposal to include fundamental fields is still such that only diagrams with a single boundary contribute. Nevertheless this could be due to multiple boundaries being considered as disconnected diagrams, and thus not contributing to the free energy.
(2) Diagrams with more than one boundary simply vanish. If this were the case, this would give a highly nontrivial prediction for the matrix model.

Whatever the cause, we believe that the impressive agreement between the gauge theory and matrix model results strongly supports the fact that matrix model diagrams with more than one boundary do not contribute to the duality, and we look forward to seeing how this phenomenon will emerge within a purely gauge theoretic framework.

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